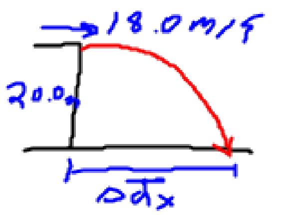
Projectile Motion

Horizontal Projectiles

1) Head-Smashed-In Buffalo Jump, near Fort Macleod, Alberta, is a UNESCO heritage site. Over 6000 years ago, the Blackfoot people of the Plains hunted the North American bison by gathering herds and directing them over cliffs 20.0 m tall. Assuming the plain was flat so that the bison ran horizontally off the cliff, and the bison were moving at their maximum speed of 18.0 m/s at the time of the fall, determine how far from the base of the cliff the bison landed. [36.3 m]



$\vec{v}_x = 18.0\ m/s$
$\Delta d_y = -20.0\ m$
$\vec{a} = -9.81\ m/s^2$
$\Delta t = ?$
$\Delta \vec{d}_x = ?$
$\vec{v}_y = 0\ m/s$

Time to fall

$\Delta \vec{d}_y = \vec{v}_1 \Delta t + \frac{1}{2} a (\Delta t)^2$

$\Delta \vec{d}_y = \frac{1}{2} \vec{a} (\Delta t)^2$

$\sqrt{\frac{2 \Delta \vec{d}}{\vec{a}}} = \Delta t$

$\Delta t = \sqrt{\frac{2(-20.0\ m)}{-9.81\ m/s^2}}$

$\Delta t = 2.0193\ s$

Range

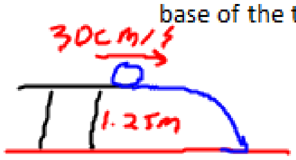
$\Delta \vec{d}_x = \vec{v}_x \Delta t$

$\Delta \vec{d}_x = (18.0\ m/s)(2.0193\ s)$

$\Delta \vec{d}_x = 36.3474\ m$

$\boxed{\Delta \vec{d}_x = 36.3\ m}$

2) A coin rolls off a table with an initial horizontal speed of 30 cm/s. How far will the coin land from the base of the table if the table's height is 1.25 m? [15 cm]



$\vec{v}_x = 30\ cm/s = 0.30\ m/s$
$\Delta \vec{d}_y = -1.25\ m$
$\vec{a}_y = -9.81\ m/s^2$
$\vec{v}_{y1} = 0\ m/s$
$\Delta t = ?$
$\Delta \vec{d}_x = ?$

Time to fall:

$\Delta \vec{d}_y = \vec{v}_{y1} \Delta t + \frac{1}{2} \vec{a}_y (\Delta t)^2$

$\Delta \vec{d}_y = \frac{1}{2} \vec{a}_y (\Delta t)^2$

$\sqrt{\frac{2 \Delta \vec{d}_y}{\vec{a}_y}} = \Delta t$

$\Delta t = \sqrt{\frac{2(-1.25\ m)}{-9.81\ m/s^2}}$

$\Delta t = 0.5048\ s$

Range:

$\Delta \vec{d}_x = \vec{v}_x \Delta t$

$\Delta \vec{d}_x = 30\ cm/s\ (0.5048\ s)$

$\Delta \vec{d}_x = 15.144\ cm$

$\boxed{\Delta \vec{d}_x \approx 15\ cm}$

3) An arrow is fired horizontally with a speed of 25.0 m/s from the top of a 150.0-m-tall cliff. Assuming no air resistance, determine the distance the arrow will drop in 2.50 s. [30.7 m]

$\vec{v}_x = 25.0\,m/s$

$v_{y1} = 0\,m/s$

$a_y = -9.81\,m/s$

$\Delta t = 2.50\,s$

$\Delta \vec{d} = ?$

$$\Delta \vec{d}_y = \vec{v}_{y1}\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\Delta \vec{d}_y = \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\Delta d_y = \frac{1}{2}(-9.81\,m/s^2)(2.50\,s)^2$$

$$\Delta d_y = -30.65625\,m$$

$$\boxed{\Delta d_y \approx -30.7\,m}$$

$\therefore$ will drop 30.7 m

4) An object is thrown horizontally off a cliff with an initial speed of 7.50 m/s. The object strikes the ground 3.0 s later. Find

a. the object's vertical velocity component when it reaches the ground [29 m/s [down]]

$\vec{v}_{y1} = 0\,m/s$

$\vec{a}_y = -9.81\,m/s^2$

$\Delta t = 3.0\,s$

$\vec{v}_{y2} = ?$

$$\vec{v}_{y2} = \vec{v}_{y1} + \vec{a}_y\Delta t$$

$$\vec{v}_{y2} = -9.8\,m/s^2(3.0\,s)$$

$$\vec{v}_{y2} = -29.4\,m/s$$

$$\boxed{\vec{v}_{y2} \approx -29\,m/s}$$

b. the distance between the base of the cliff and the object when it strikes the ground [23 m]

$\Delta \vec{d}_x = ?$

$v_x = 7.50\,m/s$

$\Delta t = 3.0\,s$

$$\Delta \vec{d}_x = v_x\Delta t$$

$$\Delta d_x = 7.50\,m/s(3.0\,s)$$

$$\Delta d_x = 22.5\,m$$

$$\boxed{\Delta d_x \approx 23\,m}$$

c. the horizontal velocity of the object 1.50 s after its release [7.50 m/s]

Neglecting air resistance, horizontal velocity does not change!

$$\therefore \boxed{\vec{v}_{x1} = \vec{v}_{x2} = 7.50\,m/s}$$

5) A baseball player throws a ball horizontally at 45.0 m/s. How far will the ball drop before reaching first base 27.4 m away? [1.82m]

$\vec{v}_x = 45.0\,m/s$

$\Delta d_y = ?$

$\Delta d_x = 27.4\,m$

$\vec{a}_y = -9.81\,m/s^2$

$\vec{v}_{y_1} = 0\,m/s$

Find time of flight:

$\vec{v}_x = \frac{\Delta d_x}{\Delta t}$

$\Delta t = \frac{\Delta d_x}{v_x}$

$\Delta t = \frac{27.4\,m}{45.0\,m/s}$

$\Delta t = 0.60\overline{8}\,s$

Find drop $\Delta \vec{d}_y$:

$\Delta \vec{d}_y = \vec{v}_{1y}\Delta t + \frac{1}{2}\vec{a}_y(\Delta t)^2$ ($\vec{v}_{1y} = 0$)

$\Delta \vec{d}_y = \frac{1}{2}\vec{a}_y(\Delta t)^2$

$\Delta \vec{d}_y = \frac{1}{2}(-9.81\,m/s^2)(0.60\overline{8}\,s)^2$

$\Delta \vec{d}_y = -1.81851\,m$

$\Delta \vec{d}_y \approx -1.82\,m$

6) An astronaut stands on the edge of a lunar crater and throws a half-eaten Twinkie™ horizontally with a velocity of 5.00 m/s. The floor of the crater is 100.0 m below the astronaut. What horizontal distance will the Twinkie™ travel before hitting the floor of the crater? (The acceleration of gravity on the moon is 1/6th that of the Earth). [55.3 m]

$\vec{v}_x = 5.00\,m/s$

$\Delta d_y = -100.0\,m$

$\Delta d_x = ?$

$\Delta t = ?$

$\vec{a}_y = \frac{-9.81\,m/s^2}{6}$

$\vec{v}_{y_1} = 0\,m/s$

Find time to Fall:

$\Delta \vec{d}_y = \vec{v}_{1y}\Delta t + \frac{1}{2}\vec{a}_y(\Delta t)^2$ ($\vec{v}_{1y} = 0$)

$\Delta \vec{d}_y = \frac{1}{2}\vec{a}(\Delta t)^2$

$\sqrt{\frac{2\Delta \vec{d}_y}{\vec{a}}} = \Delta t$

$\Delta t = \sqrt{\frac{2(-100.0\,m)}{-\frac{9.81}{6}\,m/s^2}}$

$\Delta t = 11.06\,s$

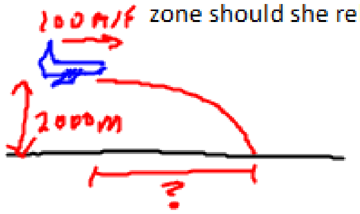
Range:

$\Delta \vec{d}_x = \vec{v}_x \Delta t$

$\Delta \vec{d}_x = (5.00\,m/s)(11.06\,s)$

$\Delta \vec{d}_x \approx 55.3\,m$

7) A rescue pilot drops a survival kit while her plane is flying at an altitude of 2000.0 m with a forward velocity of 100.0 m/s. If air friction is disregarded, how far in advance of the starving explorer's drop zone should she release the package? [2020 m]



$\Delta d_y = -2000.0\,m$
$\vec{v}_x = 100.0\,m/s$
$\vec{a}_y = -9.81\,m/s^2$
$\vec{v}_{y1} = 0\,m/s$
$\Delta t = ?$
$\Delta d_x = ?$

Time to fall:

$\Delta \vec{d}_y = v_{y1}\Delta t + \frac{1}{2} a_y (\Delta t)^2$

$\sqrt{\frac{2\Delta \vec{d}_y}{a_y}} = \Delta t$

$\sqrt{\frac{2(-2000.0\,m)}{-9.81\,m/s^2}} = \Delta t$

$\Delta t = 20.1928\,s$

Range:

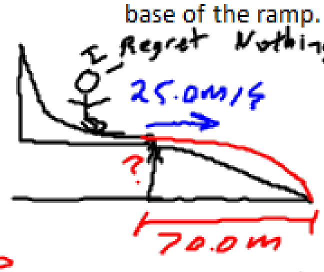
$\Delta \vec{d}_x = \vec{v}_x \Delta t$

$\Delta \vec{d}_x = (100.0\,m/s)(20.1928\,s)$

$\Delta \vec{d}_x = 2019.28\,m$

$\boxed{\Delta \vec{d}_x \approx 2020\,m}$

8) A skier leaves the horizontal end of a ramp with a velocity of 25.0 m/s [E] and lands 70.0 m from the base of the ramp. How high is the end of the ramp from the ground? [38.5 m]



$\vec{v}_x = 25.0\,m/s\,[E]$
$\Delta \vec{d}_x = 70.0\,m$
$\Delta d_y = ?$
$\Delta t = ?$
$\vec{v}_{y1} = 0\,m/s$
$\vec{a}_y = -9.81\,m/s^2$

Time in flight:

$\vec{v}_x = \frac{\Delta \vec{d}_x}{\Delta t}$

$\Delta t = \frac{\Delta \vec{d}_x}{\vec{v}_x}$

$\Delta t = \frac{70.0\,m}{25.0\,m/s}$

$\Delta t = 2.8\,s$

Height:

$\Delta \vec{d}_y = \vec{v}_1 \Delta t + \frac{1}{2}\vec{a}_y \Delta t^2$

$\Delta \vec{d}_y = \frac{1}{2}\vec{a}_y \Delta t^2$

$\Delta d_y = \frac{1}{2}(-9.81\,m/s^2)(2.8\,s)^2$

$\Delta d_y = -38.4552\,m$

$\boxed{\therefore \text{The height is } 38.5\,m}$

9) What is the horizontal speed of an object if it lands 40.0 m away from the base of a 100-m-tall cliff? [8.86 m/s]

$\vec{v}_x = ?$
$\Delta \vec{d}_x = 40.0\,m$
$\Delta d_y = -100\,m$
$\vec{v}_{y1} = 0\,m/s$
$\vec{a}_y = -9.81\,m/s$
$\Delta t = ?$

Time to fall:

$\Delta \vec{d}_y = \vec{v}_{y1}\Delta t + \frac{1}{2}\vec{a}_y \Delta t^2$

$\sqrt{\frac{2\Delta \vec{d}_y}{\vec{a}_y}} = \Delta t$

$\Delta t = \sqrt{\frac{2(-100\,m)}{-9.81\,m/s^2}}$

$\Delta t = 4.5152\,s$

Get $\vec{v}_x$ from Range:

$\vec{v}_x = \frac{\Delta d_x}{\Delta t}$

$\vec{v}_x = \frac{40.0\,m}{4.5152\,s}$

$\vec{v}_x = 8.85897\,m/s$

$\vec{v}_x \simeq 9\,m/s$

10) Participants in a road race take water from a refreshment station and throw their empty cups away farther down the course. If a runner has a forward speed of 6.20 m/s, how far in advance of a garbage pail should he release his water cup if the vertical distance between the lid of the garbage can and the runner's point of release is 0.50 m? [2.0 m]

$\vec{v}_x = 6.20\,m/s$
$\Delta \vec{d}_x = ?$
$\Delta d_y = -0.50\,m$
$\Delta \vec{v}_{y1} = 0\,m/s$
$\vec{a}_y = -9.8\,m/s$
$\Delta t = ?$

Time to fall:

$\Delta \vec{d}_y = \vec{v}_{y1}\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$

$\sqrt{\frac{2\Delta \vec{d}_y}{\vec{a}}} = \Delta t$

$\Delta t = \sqrt{\frac{2(-0.50\,m)}{-9.81\,m/s^2}}$

$\Delta t = 0.3193\,s$

Range:

$\Delta \vec{d}_x = \vec{v}_x \Delta t$

$\Delta \vec{d}_x = 6.20\,m/s(0.3193\,s)$

$\Delta \vec{d}_x = 1.97966\,m$

$\Delta \vec{d}_x \simeq 2.0\,m$